Monetary policy at the zero lower bound

Discretion, uncertainty and interest rate volatility

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ABSTRACT

This study investigates optimal monetary policy under a set of simultaneous constraints: i) when nominal interest rates are at their zero lower bound (ZLB), ii) when there is uncertainty about the natural interest rate, and iii) when the central bank cannot make credible commitments. In these situations, there are asymmetric risks in setting monetary policy, as the ZLB prevents the central bank from further decreasing nominal rates to stimulate the economy, and unconventional monetary policy is an imperfect substitute for the constrained traditional interest rate instrument. There is therefore a trade-off between keeping interest rates lower for longer, to ensure recovery, and beginning to raise rates earlier but more gradually. To assess the ability of different policies to handle this trade-off, I use a New Keynesian dynamic stochastic general equilibrium model with monetary policy defined either by a standard Taylor rule or discretionary inflation targeting that considers uncertainty. The central finding is that an optimal discretionary policy augmented to account for uncertainty about the natural interest rate forcing the ZLB to bind can perform as well or better than the Taylor rule by staying lower for longer, even though the discretionary policy has a more volatile policy rate. A further result is that a Rogoff-type conservative central banker who puts a high relative weight on stabilizing inflation can alleviate the deflationary pressure of the ZLB episode.

INTRODUCTION

A fundamental constraint faced by central banks is that they cannot make fully credible commitments to future policy actions. The resulting degree of discretion has been the status quo at central banks, but in recent years this has been complicated by two conditions: the zero lower bound (ZLB) on nominal interest rates and uncertainty about future economic activity. The objective of this study is to specify the characteristics of optimal monetary policy under these conditions in a standard New Keynesian model extended to incorporate uncertainty in the central bank's projection of the natural interest rate. The natural interest rate is a proxy for economic activity because it is the real interest rate that is consistent with stable prices and full employment.

The central problem of the ZLB is that interest rates below zero entail making lenders pay to hold their money in a bank. In this situation, the rational lender would prefer to hold their money in paper currency, thereby earning zero per cent interest instead of having to pay the bank. The availability of paper currency as an alternative investment acts as an interest rate floor and prevents central banks from pushing the policy rate—the overnight nominal interest rate at which banks lend to each other—significantly below zero. This dynamic is known as a liquidity trap, since the conventional policy of expanding liquidity through monetary easing becomes impotent (Krugman, 1998; Eggertsson, 2006; Svensson, 2006).[1]

In weak economic times, the central bank's primary tool is to lower the policy rate to stimulate output and inflation. As the policy rate approaches zero, the ZLB hinders economic recovery by preventing central banks from decreasing it any further. This barrier makes it particularly difficult to combat persistently low inflation, and the prospect of interest rates at the ZLB can even induce deflation—a widespread decline in prices. The ZLB has proved a serious constraint for the Bank of Japan since the late

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[1] A stricter definition is that an economy is not in a liquidity trap until the central bank has bought enough of the economy's interest-bearing assets to push all of their interest rates to zero. As put by Orphanides (2003, p. 19), "additional monetary expansion continues to have some bite because the prices and yields of all assets, not merely 'the' short-term nominal rate of interest, jointly determine aggregate demand." For the purposes of this paper, I maintain the definition associated with the central bank policy rate being at the ZLB.

1990s and for many other central banks since the financial crisis of 2007-2008, notably including the Federal Reserve and the European Central Bank.

Central banks have employed unconventional monetary policy as an alternative means of affecting interest rates when the policy rate is at the ZLB; these unconventional policies include large-scale asset purchases (LSAPs), forward guidance, and slightly negative policy rates. However, the ZLB remains a serious constraint because these tools are imperfect substitutes for the traditional policy rate. Firstly, the benefits of LSAPs for the wider economy are very difficult to estimate, and there are a series of associated costs that limit the extent of feasible asset purchases (Bernanke, 2012). These include risks to financial instability, the impairment of certain asset markets, *beggar-thy-neighbour* currency devaluations, and equity concerns.

Secondly, negative policy rates, though possible, cannot go below approximately negative 0.5 per cent, as the costs to holding capital as paper currency are not infinite. Therefore, at some point agents will simply choose to invest in vaults, rather than pay exceedingly negative interest rates (Bank of Canada, 2015). Lastly, forward guidance about intended policy actions is not a perfect substitute for actual policy rate changes because central banks are unable to commit credibly beyond the short term. This inability stems from the fact that there are changes in economic conditions, our understanding of the economy, and central bank leaders (Woodford, 2003).

The result of the ZLB hindering the first-best policy tool is an asymmetry in policy risks in the central bank's decision-making process. The relative inadequacy of unconventional policy responses means that central banks should give higher consideration to downside risks such as deflation and recession near the ZLB, even if the expected values of the upside and downside risks are the same. Upside risks, such as unexpected increases in output and inflation, can be countered more easily with a rapid increase of the policy rate to an appropriate level—a well-practiced policy action.

Assuming that central banks can commit to future actions via forward guidance is instructive for studying the theoretically optimal policy, but this assumption must be relaxed to accord with reality. A discretionary central bank has much more difficulty combating the ZLB because it cannot commit itself to a post-ZLB burst of inflation (Krugman, 1998). Theories of optimal monetary policy assume central banks can fully commit to certain policies and that there is no uncertainty in the projections of key variables by the central bank.[2] Furthermore, some studies of these theories do not include a non-negativity constraint representing the ZLB (Rotemberg and Woodford, 1997; Schmitt-Grohe and Uribe, 2004). While normally there are higher welfare losses under discretion relative to commitment, these losses are even higher at the ZLB (Adam and Billi, 2005).

Evans *et al.* (2015) model a discretionary central bank facing the explicit ZLB constraint and uncertainty about whether the natural rate of interest forces the ZLB to bind. Their result is that the optimal discretionary policy postpones raising the policy rate from the ZLB by two to three quarters, and then raises it more steeply, all relative to policies that ignore uncertainty. A counter-argument to this relatively delayed rate increase is that the sudden changes in the policy rate cause social welfare losses through volatility in financial markets (Stein and Sunderam, 2015). I extend the Evans *et al.* (2015) model to assess this trade-off, and I focus on discretionary policy at the ZLB because the commitment case is well studied and because the discretionary case is a better depiction of reality for most central banks.

First, I augment the welfare criterion, used to judge the desirability of alternative policies, to assess whether a policy rate path that remains “lower for longer” is excessively volatile when it finally leaves the ZLB. The main finding is that optimal discretionary policy does not impose undue welfare losses through pol-

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[2] See Woodford (2003) for an authoritative review of both optimal commitment and discretion theory.

icy-rate volatility, unless society both puts a disproportionately high value on stabilizing the policy rate and cannot hedge against such volatility. Second, given the deflationary bias of purely discretionary policy at the ZLB, I introduce a so-called *Rogoff conservative* central banker that places a much higher value on inflation stabilization than output gap stabilization. The Rogoff conservative central banker is helpful at increasing inflation back to target during a liquidity trap, with the caveat that there are is high policy rate volatility.

THEORETICAL FRAMEWORK

The liquidity trap as a credibility problem

During a liquidity trap, the nominal policy rate cannot be decreased to match decreases in the natural rate into negative territory. The ZLB technically binds when the natural rate is so negative that it is lower than the negative of the inflation target; that is, when the natural rate is less than the lowest possible real interest rate (Woodford 2003). The resulting gap of the real rate above the natural rate slows the economy and decreases inflation. A vicious cycle then begins, as decreases in inflation increase the real rate still further above the natural rate.

Krugman (1998) articulated the crux of the liquidity trap as a credibility problem: for an expansion of the money supply to raise prices in equal proportion, the central bank must commit to the policy credibly enough to convince the market that the volume of the expansion will not be reversed in the future. The market, however, expects the discretionary central bank to renege and prevent the inflationary burst as soon as the ZLB is no longer binding. Commitment to this post-ZLB boom would, if credible, increase welfare because it would cause lower long-term real interest rates even while the policy rate is stuck at zero. That effect would feed back into a reduced incentive to save during the liquidity trap, thereby spurring output and preventing price cuts in a "virtuous circle" (Woodford, 2010, p. 753). Nevertheless, for discretionary central banks this promise is not credible, and the welfare loss from discretion relative to commitment is even higher at the ZLB (Adam and Billi, 2005).

The canonical New Keynesian model

The theoretical framework I use to analyse the optimal monetary policy problem is the canonical New Keynesian sticky-price model considered by Clarida, Galí, and Gertler (1999) and Woodford (2003), *inter alia*. This framework includes forward-looking behaviour and is derived from micro-foundations of optimization by households and firms.[3] The temporary nominal rigidities arise from price-setting behaviour as outlined by Calvo (1983).

I begin with the two central equations of the model. Let y_t be the logarithm of output and z_t be the logarithm of the potential level of output, the latter being the level of output that would occur in the case of fully flexible wages and prices. The logarithm of the output gap is defined by $x_t = y_t - z_t$. Furthermore, let π_t be the inflation rate, equalling the per cent change in the price level from period *t*-1 to period *t*. Let i_t be the short-term nominal policy interest rate chosen by the central bank, and each period is taken to be a quarter. The intertemporal "*IS*" curve is the aggregate-demand relation:

$$x_t = E_t x_{t+1} - {}^1\!/\!_\sigma \, (i_t - E_t \pi_{t+1} - \rho_t^{\,n}),$$

where E_t is the expected value operator denoting rational expectations held in period *t*. The inverse elasticity of intertempo-

[3] See Bernanke, Gertler, and Gilchrist (1999) for the derivation of the micro-foundations.

ral substitution σ corresponds to the interest elasticity of the IS curve. The natural rate of interest ρ_t^n is given by:

$$\rho_t^n = \bar{\rho} + \sigma g_t + \sigma E_t (z_{t+1} - z_t),$$

where $\bar{\rho} > 0$ is the constant long-run real equilibrium interest rate, g_t is a demand shock, and z_t is again the logarithm of potential output. The result is that ρ_t^n is exogenous because both g_t and z_t are exogenous. According to the IS curve representation of expenditure decisions, higher expected output raises current output because agents smooth consumption over time. Note that $i_t - E_t\pi_{t+1}$ is equal to the period t real interest rate and that $i_t - E_t\pi_{t+1} - \rho_t^n$ is the deviation of the real interest rate from the economy's natural rate of interest, sometimes called the interest rate gap. Hence, the output gap is inversely related to positive interest rate gaps.

The aggregate-supply relation is the New Keynesian Phillips Curve (NKPC) specified as:

$$\pi_{\mathrm{t}} = \kappa x_t + \beta E_t\pi_{t+1} + u_t,$$

where κ is the slope of the NKPC, determined by the level of price stickiness in the economy and satisfying $\kappa > 0$. A higher value of κ implies less stickiness, meaning a higher portion of a given output gap is passed through into inflation. The discount factor β, with $0 < \beta < 1$, indicates how much the representative household discounts the future relative to today. The final term u_t is a cost-push shock that embodies exogenous changes in inflation. These include changes in inflation expectations, oil prices, dollar appreciation, and nominal wages that "push real wages away from their 'equilibrium' values due to frictions in the wage contracting process" (Evans *et al.*, 2015; Clarida, Galí, and Gertler, 1999, p. 1667).

The central bank gauges the desirability of a given policy by how well it minimises the welfare losses associated with deviations of inflation and output from their targets. The loss function for a given time period is:

$$L_t = (\pi_t - \pi^*)^2 + \lambda(x_t - x^*)^2,$$

where π^* is the target inflation rate, x^* is the logarithm of the optimal output gap, and the parameter $\lambda > 0$ is the weight on output-gap variability relative to inflation variability. These squared deviations of inflation and the output gap from their socially desired levels represent welfare costs to the representative household, which the central bank tries to minimise.[4] Without loss of generality, we can assume that the steady-state inflation target and optimal output gap are zero, so that the full intertemporal loss function becomes:

$$L = ½\, E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2),$$

where the discount factor β^t is the same as in the NKPC.[5] The central bank will prefer the policy that it expects to minimise the discounted welfare losses.

[4] Woodford (2003) derives this loss function from welfare-theoretic foundations as the negative of a quadratic approximation of the utility of the representative household.

[5] However, if we take the intertemporal loss function to represent a central bank's preferences or its legislative mandate, then the discount factor may diverge from that of the representative household (Woodford, 2014).

METHODOLOGY

Numerical simulations

I perform numerical simulations in a New Keynesian dynamic stochastic general equilibrium model founded in the optimal policy framework discussed in Section 2. It is a forward-looking model assuming rational expectations that I solve for optimal discretionary policy as well as under commitment to variants of the Taylor rule, a method to model a central bank's public commitment to maintaining a particular monetary policy. The model is solved around a zero-inflation steady state, implying that the ZLB is binding if the natural rate is negative. I extend the model programmed in MATLAB by Evans *et al.* (2015) in order to include natural interest rate shocks and the calculation of welfare losses with a different loss function. It is a closed-economy model and no fiscal policy shocks are assumed, although they can be incorporated as aggregate demand shocks. Given that unconventional monetary policy is an imperfect substitute for the policy rate, I abstract from large-scale asset purchases and assume that the ZLB is a hard constraint.

Since the central bank I study here is discretionary, it cannot do "Odyssean" forward guidance, in which the central bank publicly commits itself to specific future interest rate movements in the future. A discretionary central bank cannot credibly tie its hands to prevent itself later reneging, so financial markets will not believe any such promises about the future. Rather, the central bank engages in "Delphic" forward guidance, in which it publicly states forecasts of the economy and likely policy (Campbell *et al.*, 2012). Effectively, the central bank publishes its forecasts of inflation, GDP growth, and how it would conduct monetary policy if and when such forecasts turn out to be correct. Since this model is forward-looking and solved by backward induction from a known terminal steady state, the assumption of "rational expectations" entails perfect private-sector foresight. Policy paths are anticipated, as are shocks to the economy. These simulations should therefore be interpreted in the spirit of Laséen and Svensson's (2011) description of anticipated policy paths. They are equilibria that "correspond to situations where the central bank transparently announces that it, conditional on current information, plans to implement a particular policy-rate path and where this announced plan for the policy rate is believed and then anticipated by the private sector" (Laséen and Svensson, 2011, pp. 1-2).

For convenience, I repeat the model here in the minimization form:

$$\min_{it} \tfrac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^{\,2} + \lambda x_t^{\,2})$$

subject to:

$$x_t = E_t x_{t+1} - \tfrac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho_t^{\,n}),$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t,$$

$$i_t \geq 0.$$

Since the central bank is discretionary, and its policy decisions do not affect future losses, it simply minimises $\pi_t^2 + \lambda x_t^2$ each period. The possibility of a slightly negative policy rate is ruled out by the non-negativity constraint on the policy rate.

For the numerical simulations, I follow the solution methods of Evans *et al.* (2015), both for the discretionary policy and the Taylor rules.[6] Their parameters for the slope of the Phillips

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[6] For the solution methods of Evans et al. (2015), see their Appendix 1 here: https://sites.google.com/site/fgourio/research_chrono/publications.

curve κ, the inverse elasticity of intertemporal substitution σ, and the discount factor β "are all set to values common in the New Keynesian literature" and therefore maintained in the present study (Evans *et al.*, 2015, pp. 159). Appendix A (online) contains the full list of parameter values. I calibrate the model to reflect the state of the economy of the United States, with $t = 0$ being the first quarter of 2016, and the simulations occur over six years.

The assumptions about the exogenous variables are as follows. After the last date, $t = 24$, there is no uncertainty in the model. Therefore, for $t \geq 24$, the cost-push shock $u_t = 0$, and the natural rate ρ_t^n equals the long-run equilibrium real rate $\bar{\rho} = 1.3$ per cent. The value of $\bar{\rho}$ corresponds to the Federal Reserve's median long-run policy rate from March 2016 minus the Federal Reserve's inflation target $\pi^* = 2$ per cent. For $t < 24$, the natural rate ρ_t^n has a deterministic part and a random part:

$$\rho_t^n = f_t + \varepsilon_t$$

The deterministic part f_t rises linearly toward $\bar{\rho} = 1.3$ per cent. Also for $t < 24$, the cost-push shock u_t is an exogenous mean-zero stochastic process.[7] I keep the calibration of Evans *et al.* (2015) for the initial natural rate of $\rho_1^n = -0.5$ per cent, which is similar to the estimates in Laubach and Williams (2015, Figures 5 and 10).

From the perspective of period t, the central bank observes both the value of the current natural rate ρ_t^n and cost-push shock u_t. Since the starting natural rate is negative, the ZLB initially binds, and the question is when to lift the policy rate from zero. The model is solved by backward induction with 50,000 simulations drawn from the distributions of the stochastic elements.

[7] The natural rate's random component ε_t is an AR(1) stochastic process with a standard deviation of innovation $\sigma_\varepsilon=1.32$ and a serial correlation of $\rho_\varepsilon=0.85$. The cost-push shock u_t is also an AR(1) stochastic process with a standard deviation of innovation $\sigma_u=0.1$ and a serial correlation of $\rho_u=0$. The cost-push shock is independent and identically distributed. See Evans *et al.* (2015, p. 19) for a discussion of this parameterization.

Policy reaction functions

The key addition Evans *et al.* (2015) make is allowing the central bank to consider uncertainty about the natural rate and, hence, the probability of the policy rate being constrained by the ZLB. This change is a departure from the certainty equivalence assumption in many models, in which the environment is assumed to be deterministic, and optimal policy responds only to the mean of the target variables instead of considering the distribution of future disturbances (Evans *et al.*, 2015; Woodford, 2003). If the natural rate is negative, then the policy rate is set to zero. If the natural rate is positive, however, the optimal path for the policy rate is given by this reaction function:

$$i_t = \rho_t^n + E_t\pi_{t+1} + \sigma\,(E_t x_{t+1} - x_t).$$

This reaction function is an example of what Svensson (2003) calls an implicit reaction function, as it is a relationship between the policy rate and endogenous, non-predetermined variables. In the context of the full model, this function is an equilibrium condition that must be "solved together with the rest of the model in order to determine the instrument setting" (Svensson, 2003, pp. 14). While the current natural rate ρ_t^n is observed by the central bank, the other variables on the right-hand side depend on unknown expected values of future conditions of the economy and the policy rate. Specifically, uncertainty about the natural rate enters the policy rate decision because x_t, $E_t x_{t+1}$, and $E_t\pi_{t+1}$ all depend on uncertain future values of the natural rate.

To account for the risk of a negative natural rate and a binding ZLB from this uncertainty, Evans *et al.* (2015) model risk management by truncating the distribution of the natural rate when they calculate the expected values of endogenous variables.

Hence, the endogenous variables are only affected by the negative part of the distribution of the natural rate. I maintain this treatment of the asymmetric risk because a positive natural rate in future periods can be offset simply by traditional increases in the policy rate.

A small but instructive addition to this optimal discretionary policy is to assume that the policymaker is a "Rogoff conservative" central banker. In this case, the central bank puts significantly higher weight on stabilizing the inflation gap rather than the output gap, and the private sector understands this (Rogoff, 1985). While this method was originally developed to tackle the positive cost-push shocks of the 1970s that created high inflation, it is also useful in a liquidity trap. Essentially, restoring inflation from below target is very important for lowering long-term real interest rates and spurring consumption today. I implement this by decreasing the relative weight λ on the output gap in the loss function that is used for deriving the optimality conditions from $\lambda = 0.25$ to a lower Rogoff-style value of $\lambda_R = 0.1$. Even though λ_R is less than the "true" λ in society's loss function, this change can yield better overall outcomes by effectively changing how the private sector expects the central bank to respond to disturbances.

I follow Evans *et al.* (2015) in comparing the optimal discretionary policy with three other reaction functions. The first is a "naïve" policy that is the same as optimal discretion, except the central bank does not consider uncertainty about the future. The private sector knows this but still considers uncertainty itself. The central bank is then surprised in two ways: the arrival of each new shock and the realised values of π_t and x_t, which the private sector determined with rational expectations.

The two reaction functions representing the commitment case are both forms of the Taylor rule, a formula the central bank publicly commits to use for setting the policy rate systematically. Such a scenario is instructive for comparison with the discretionary case because it models the economy's evolution under the extreme assumption of fully credible commitment. The first Taylor rule is the original Taylor (1993) formulation with the constant long-run equilibrium real interest rate $\bar{\rho}$, and the second reacts to the short-term natural rate ρ_t^n:

Taylor rule with constant LR rate (TRCI):

$$i_t = \bar{\rho} + \pi^* + \phi\, \pi_t + y x_t$$

Taylor rule with time-varying natural rate (TRTV):

$$i_t = \rho_t^n + \pi^* + \phi\, \pi_t + y x_t.$$

The weights on the inflation gap ϕ and the output gap y are set to 1.5 and 0.5 respectively, as in Taylor (1993). The other notation remains the same as above. Apart from the constant long-run equilibrium real interest rate $\bar{\rho}$ and constant inflation target π^*, both the TRCI and TRTV are set by the central bank immediately in response to that period's predetermined variables. The weights imply that the policy rate rises by 1.5 times the inflation gap and 0.5 times the output gap. The TRTV's one-to-one response with the short-term natural rate ρ_t^n means that it responds more accurately to the level of activity in the economy, as opposed to the TRCI's use of the unchanging long-run equilibrium real interest rate $\bar{\rho}$.

To assess concerns of welfare losses from policy rate volatility, I run the same simulations again with a standard addition to the loss function to penalise changes in the policy rate (Woodford, 2003). This addition is useful for determining whether an otherwise- desirable policy only stabilises inflation and output through intolerably high variation in the policy rate. The per-period loss function becomes:

$$L_t = (\pi_t - \pi^*)^2 + \lambda(x_t - x^*)^2 + \lambda_{\Delta i}(i_t - i_{t-1}).$$

The parameter $\lambda_{\Delta i}$ is the weight placed on changes in the policy rate between periods $(i_t - i_{t-1})$. This extra term in the loss function indicates that the central bank has a preference for interest rate smoothing. The weight $\lambda_{\Delta i}$ is usually set to 0.1 or 0.5 in the literature (Rudebusch, 2006; Alichi *et al.*, 2015). Although aversion to changes in the policy rate does not enter society's true loss function, adding this term into the central bank's loss function can be motivated by a desire to stabilise the real economy as well as observed smoothing by central banks.

For instance, Brainard (1967) demonstrates that gradual changes in the policy rate are beneficial when there is uncertainty about how the policy will affect the economy, and high volatility in bond markets could hinder financial intermediation and the credit-supply process (Alichi *et al.*, 2015). Woodford (2003) also argues that more gradual monetary policy increases its predictability, allowing small changes in the policy rate to cause large changes in long-term interest rates. The problem then follows that central bank gradualism may not prevent volatility in longer-term interest rates because a given change in the policy rate is amplified when interpreted by the markets as a signal of future changes to come (Stein and Sunderam, 2015).

Rudebusch (2006) gives two arguments for smaller weights on policy rate changes. First, the United States had high interest rate volatility during the 1979-82 monetary experiment, and this did not generate considerable costs on its own. Second, the weights of $\lambda_{\Delta i} = 0.1$ or 0.5 "still seem at the high end of the plausible range of penalties to reduce volatility, especially in a world with a wide variety of financial market instruments that allow for hedging against interest rate volatility" (Rudebusch, 2006, p. 107). With this disclaimer in mind, I report estimates with both weights.

SIMULATION RESULTS

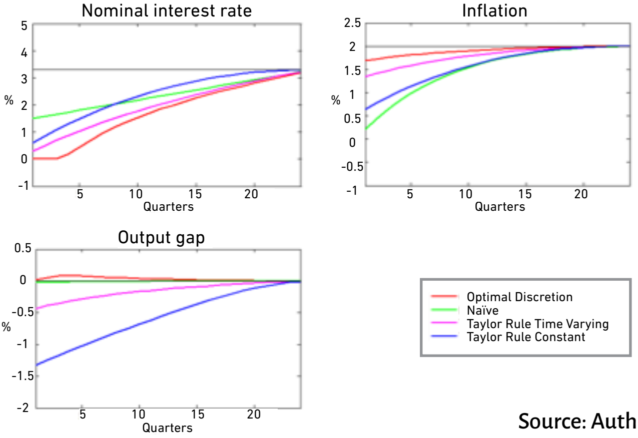
Figure 1 displays the modal outcomes for the evolution of the economy in the baseline re-calibrated model starting in the first quarter of 2016.[8] The black horizontal lines are the steady-state values of each variable. For the policy rate, the horizontal line is the long-run policy rate projected by the Federal Reserve, which is equal to the long-run equilibrium real rate $\bar{\rho} = 1.3$ per cent plus the inflation target of 2 per cent. Note that the plot in which the optimal discretionary policy uses the Rogoff conservative weight in the loss function is reported in Appendix B (online). There are no shocks in the baseline simulation.

The optimal discretionary policy "lifts off" from the ZLB later than the other policies. The naïve policy simply tracks the natural rate's increase throughout the simulation, so the difference between the green and red interest rate paths illustrates how uncertainty about the ZLB binding impacts discretionary policy. This difference is between 0.5 and 1.6 percentage points over the first two years. The inflation outcomes for the naïve policy are much worse because it maintains a high policy rate early, but the output gap outcomes under the optimal discretionary and naïve policies are very similar. Of the two Taylor rules, the one with the time-varying natural rate ρ_t^n (TRTV) performs much better than the original formulation (TRCI) of Taylor (1993), which includes the constant long-run equilibrium real rate $\bar{\rho}$. Since $\bar{\rho}$ is by definition larger than ρ_t^n here, and the rules are otherwise identical, it is clear that the TRTV will prescribe a more gradual increase. The poor inflation and output gap performance of the TRCI is due to

[8] In all graphs here, I follow Evans *et al.* (2015) in reporting the modal outcome; this measure is useful because the Federal Open Market Committee members report their modal forecasts of monetary policy in their Summary of Economic Projections.

the fact that the private sector anticipates the higher future policy rate, and this dampens consumption and inflation today.

FIGURE 1 EVOLUTION OF KEY VARIABLES



Source: Author.

Table 1 Simulation results

Statistic	Optimal Discretion	Opt. Disc. Rogoff	Naïve	Taylor Rule (TRCI)	Taylor Rule (TRTV)
Loss	0.03	0.03	0.12	0.14	0.04
Loss + 0.1 $(i_t - i_{t-1})$	0.10	0.14	0.18	0.17	0.10
Loss + 0.5 $(i_t - i_{t-1})$	0.39	0.56	0.41	0.29	0.34
Mean time to liftoff	4.63	5.16	1	1	1
Median time to liftoff	3	3	1	1	1
Median π at liftoff	1.72	1.65	0.21	0.64	1.34
Median x at liftoff	0.05	0.02	-2.19	-1.33	-0.43
75th percentile of max π	2.64	2.55	2.27	2.43	2.05
25th percentile of min x	-0.91	-1.04	-2.19	-2.51	-1.10
Median std. dev. of Δi	1.89	3.06	1.22	1.04	1.39

With "liftoff" defined as the policy rate being increased above 0.25 per cent, all policies other than optimal discretion and its modified Rogoff formulation leave the ZLB in the first quarter. Optimal discretion does not raise the policy rate until a median of the 2 quarters or a mean of 3.63 quarters later than the other policies. In the Evans *et al.* (2015) study, the analogous delay at the ZLB under optimal discretion is two to three quarters after the other policies rise. The extra delay in this simulation is due to the fact that the long-run projected policy rate has been adjusted downward from 3.5 to 3.3 per cent, reflecting weaker aggregate demand than expected. With the modification for the Rogoff conservative central banker, optimal discretion performs largely similarly, with two exceptions. Firstly, it's mean time for raising the policy rate is even later, reflecting the central bank's desire to restore inflation to target quickly. Secondly, it has higher volatility in interest rate changes.

The welfare losses in Table 1 show that optimal discretion performs very well according to the original loss function, incurring 1/4 the loss of the naïve policy, 1/5 the loss of the TRCI, and 3/4 the loss of the TRTV. With a weight of $\lambda_{\Delta i} = 0.1$ on interest rate variation, the extra losses imposed by optimal discretion put it on par with the TRTV, while the other policies still perform much worse. However, the ranking changes with the higher weight of $\lambda_{\Delta i} = 0.5$. The TRCI minimises losses because it is the least volatile—with a median standard deviation of the change in the interest rate of 1.04. For reference, Evans *et al.* (2015) empirically observe this standard deviation of changes in the Federal Reserve's policy rate to be 0.88.

Two caveats pertain to this analysis of losses from policy rate variation. First, the results are clearly sensitive to the assumed weight $\lambda_{\Delta i}$. Second, policies with a higher initial policy rate have the advantage of needing smaller policy rate changes to reach the long-run nominal policy rate of 3.3 per cent. In the first quarter of 2016, the Federal Reserve's policy rate was at a corridor of

0.25-0.50 per cent, so a policy with a greater starting value than 0.50 per cent underestimates the losses here.

Lastly, the policies balance output and inflation risks differently. Across simulations the 75th percentile of maximum inflation is only slightly higher under optimal discretion than under the other policies, and the 25th percentile of the minimum output gap under optimal discretion is slightly less than the TRTV and less than half that of the other policies.

CONCLUSIONS

The central theoretical contribution of this paper is that an optimal discretionary policy accounting for uncertainty about the natural rate forcing the ZLB to bind can perform as well or better than commitment to a Taylor rule, even though the discretionary policy has a more volatile policy rate. The validity of this finding depends on the true costs to society from changes in the policy rate and our ability to hedge against them. If these costs are high relative to the benefits of the "lower for longer" policy in resolving the liquidity trap, then a more gradual policy that begins raising rates earlier is desirable. An addendum to this finding is that the appointment of a Rogoff conservative central banker may be beneficial for discretionary central banks at or near the ZLB. While the Rogoff formulation is associated with keeping inflation expectations from running too high, a central bank that is symmetric about its inflation target will also diminish the deflationary bias of discretion at the ZLB. This last condition should not be taken for granted, as lately some central banks appear willing to tolerate negative inflation gaps more than positive ones of equal magnitude. Nonetheless, the cost from Rogoff's formulation is variation in the output gap and policy rate.

It is important to note limitations of this study before applying the findings to the real world. The model does not include unconventional policy in the form of large-scale asset purchases, "Odyssean" forward guidance, or negative policy rates. These omissions overstate the severity of the ZLB and portray it as a harder constraint than it is in reality. If the model were changed to include them, the effective lower bound would be slightly negative, liftoff would be earlier, and the deflationary effects of the liquidity trap would be dampened. Fiscal policy expansion is another tool that can support aggregate demand and help end a liquidity trap (Krugman 1998). I omit fiscal policy from the analysis both for parsimony and the fact that fiscal policy is subject to the political process, which means it is not a readily available tool for economic policymakers. The model has no international sector, which leaves out another important part of the policy decision. Inclusion of an international sector representing the economic context of downward growth forecasts in early 2016 heightens the risks of the ZLB. Lastly, the assumption of perfect foresight stemming from rational expectations likely does not accurately describe times of such unprecedented monetary policy, as expectations cannot now be informed by past experience.

As I have calibrated the model to conditions in the United States, I discuss the policy implications of this study in the context of the Federal Reserve. The implications, however, are instructive for the various central banks with policy rates still at the ZLB. The Federal Reserve is a worthy case to study not only due to the dominance of the dollar but also due to its December 2015 decision to raise the policy rate from the zero lower bound.

In September 2015, the Federal Reserve did not raise rates despite previous forward guidance that had seriously opened the possibility, and the Federal Reserve's subsequent rhetoric may have boxed it into the December 2015 decision to raise the policy rate corridor from 0.0-0.25 per cent to 0.25-0.50 per cent (Feroli *et al.*, 2016). The 2015 median personal consumption expenditures inflation of 1.3 per cent, also published during the same December 2015 meeting, was substantially below the central bank's 2 per cent inflation target. Despite its failure to achieve the 2 per cent inflation target, the Federal Reserve has made much better progress on its employment objective since the financial crisis. The unemployment rate was 4.9 per cent during the first two months of 2016. There seems to be some room for improvement here too, however, as the labour force participation rate is still not back to its pre-crisis level (Appendix C, online).

From the perspective of this study's numerical simulations, the Federal Reserve raised rates slightly early. One would expect this given that my calibration uses their updated March 2016 projection of the long-run equilibrium real interest rate that was revised downward from the December 2015 estimate. Of course, the Federal Reserve cannot know the future; the slowdown in China, and emerging markets in general, as well as further declines in the oil price contributed to the downward revision of U.S. growth and inflation projections from December 2015 to March 2016 (FOMC, 2016). In light of these developments, the Federal Reserve has not raised rates again yet (through August 2016). Chair Janet Yellen's speech at the end of March 2016 includes separate sections for risks to the real economy and the inflation outlook. Indeed, with a title of "The Outlook, Uncertainty, and Monetary Policy" the speech indicates a movement toward risk management in the spirit of Evans *et al.* (2015), as opposed to the more calendar-based forward guidance employed since the crisis (Feroli *et al.*, 2016).

In sum, this study implies a need for heightened risk management near the ZLB and a lower policy rate until the recovery advances further and the risk asymmetry moderates. Concerns about the volatility of a policy that remains at the ZLB longer are not supported by the model, nor are concerns that such a policy would be unable to respond to a burst of inflation. Lastly, the public perception of a Rogoff conservative central banker who cares equally about stabilizing negative and positive inflation gaps helps lift inflation expectations toward target.

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APPENDICES

Appendix A: Parameter values

Appendix A Parameter values

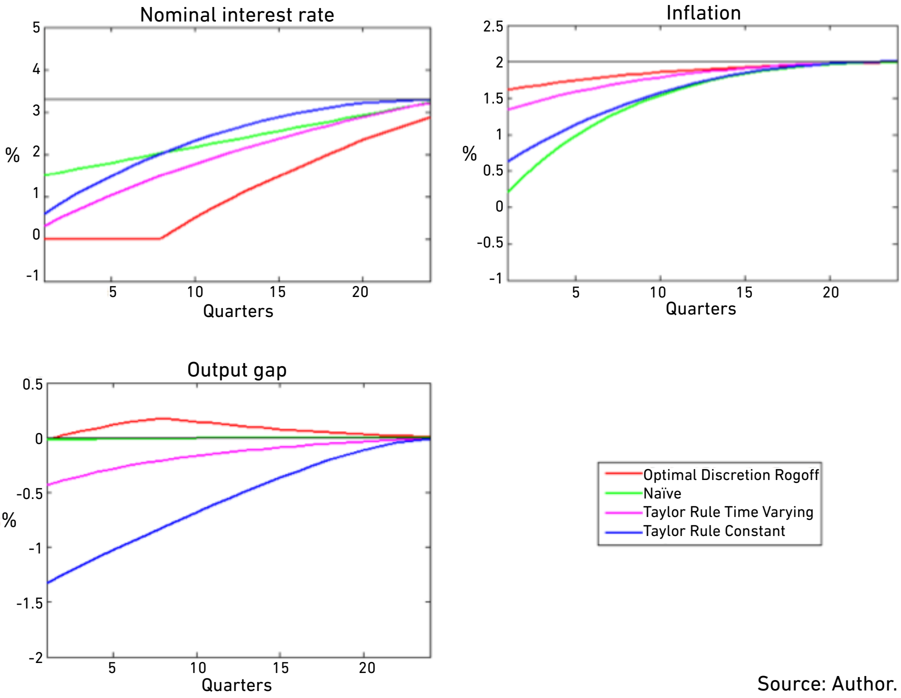
Parameter	Description	Value
β	Discount factor	0.995
$\varkappa$	Slope of Phillips Curve	0.025
σ	Inverse elasticity of substitution	2
σ_{ε}	Std. dev. of natural rate innovation	1.32
σ_{u}	Std. dev. of cost-push shock	0.10
ϱ_{ε}	Serial correlation of natural rate	0.85
ϱ_{u}	Serial correlation of cost-push	0
λ	Weight on output stabilization	0.25
π^{*}	Steady-state inflation (annualized)	2
$\varrho^{n}{}_{1}$	Value of natural rate at $t = 1$	-0.5
T	Quarters to reach terminal natural rate	24
ϱ	Terminal natural rate (annualized)	1.3
φ	Taylor rule coefficient on inflation	**1.5**
γ	Taylor rule coefficient on output gap	**0.5**

Note: The values of standard deviations, inflation, the output gap, and the natural rate are in percentage points.

Source: Author.

Appendix B: Results with Rogoff central banker implementing optimal discretion

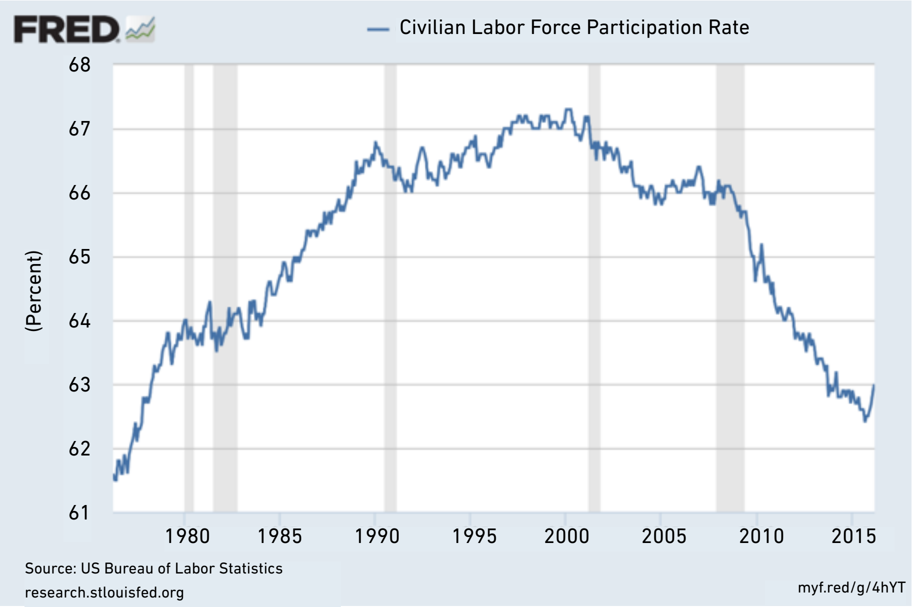
APPENDIX B RESULTS WITH ROGOFF CENTRAL BANKER IMPLEMENTING OPTIMAL DISCRETION



Source: Author.

Appendix C: United States labour force participation rate

APPENDIX C UNITED STATES LABOUR FORCE PARTICIPATION RATE



Source: St. Louis Federal Reserve Economic Data.